Sailing yacht performance in calm water and in waves.

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SAILING YACHT PERFORMANCE IN CALM WATER AND IN WAVES

by

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Summary

The Delft Systematic Yacht Hull Series has been extended to a total of 39 hull form variations, covering a wide range of length-displacement ratio's and other form parameters. The total set of modelexperiment results, including upright and heeled resistance as well as sideforce and stability, has been analysed and polynomial expressions to approximate these quantities are presented. In view of the current interest in the performance of sailing yachts in waves the added resistance in irregular waves of 8 widely different hull form variations has been calculated. Analysis of the results shows that the added resistance in waves strongly depends on the product of displacement-length ratio and the gyradius of the pitching motion.
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Nomenclature

$A_w$ - waterline area
$A_x$ - maximum cross-section area
$AR$ - aspect ratio
$B_{WL}$ - waterline breadth
$B_{MAX}$ - maximum beam
$C_F$ - frictional resistance coefficient
$C_H$ - heeled resistance coefficient
$C_M$ - maximum cross-section coefficient
$C_P$ - prismatic coefficient
$C_{Di}$ - induced resistance coefficient
$C_L$ - lift coefficient
$F_H$ - side force
$F_n$ - Froude number
$GM$ - metacentric height
$g$ - acceleration due to gravity
$H_{1/3}$ - significant wave height
$k_{yy}$ - pitch gyroradius

$LCB$ - longitudinal center of buoyancy in $\% L_{WL}$

$L_{WL}$ - waterline length

$MN$ - residuary stability

$q$ - stagnation pressure - $\frac{1}{2} \rho V^2$

$R_p$ - total resistance with heel and leeway

$R_T$ - total resistance in upright position

$R_F$ - frictional resistance

$R_R$ - residuary resistance

$R_i$ - induced resistance

$R_H$ - resistance due to heel

$R_{AW}$ - added resistance in waves

$S_k$ - wetted area keel

$S_C$ - wetted area canoe body

$S_r$ - wetted area rudder

$S_\gamma$ - spectral density

$T_1$ - wave period $T_1 = 2 \pi m_0/m_1$

$T_e$ - period of encounter

$T_E$ - effective draught

$T$ - total draught

$T_C$ - draught of canoe body

$V$ - speed

$\zeta_a$ - wave amplitude

$\phi$ - heel angle

$\lambda$ - wave length

$\rho$ - density of water

$\omega$ - circular frequency

$\nu_C$ - volume of displacement

$\Delta$ - weight of displacement

$\beta$ - leeway angle

$\mu_W$ - wave direction

$\nu$ - kinematic viscosity
1. Introduction

The research on systematic variations of sailing yacht hull forms at the Delft Ship Hydromechanics Laboratory has been extended and completed with model tests of an additional series of eleven hull form variations: Series III. The total series now consists of thirty nine models. The experimental results of the last eleven models have been used to increase the reliability of the upright resistance prediction for light displacement yachts, in particular in the high speed range with $Fn > 0.45$.

The total experimental result of the completed series has been reanalysed, also with regard to sideforce generation, stability and induced resistance. Three modifications of the keel depth of the parent model 1 have also been included in the analysis.

The resistance-speed characteristics of light-displacement yachts for speeds exceeding $Fn = 0.45$ is quite different as compared with medium and heavy-displacement yachts as shown in [1]. Consequently velocity calculations based on the results of models 1-22 (Serie I) for light-displacement yachts are not correct for the speed range where the vertical hydrodynamic lift forces on the hull cannot be neglected. For instance the IMS approximation of upright resistance, which depends to a large extend on the Delft Series I and II results, seems to underestimate the upright resistance in the high speed range as shown in Figure 1, where the residuary resistance of model 25, as calculated by the IMS and the Delft formulations are compared with experimental results [2].

The stability of a sailing yacht at heel angles up to 30 degrees is important in view of the sailcarrying capacity. In most cases the hydrostatic stability, assuming an undisturbed free surface, can be used as an approximation in a velocity prediction calculation. However, in the case of light-displacement hull forms, with a high beam to draught ratio $B_{WL}/T_c$, the distortion of the free surface and the corresponding distribution of the hydrodynamic pressure on the hull is quite different form this assumption. A stability reduction of some 20 to 30 % as compared with a hydrostatic calculation has been observed in certain cases, leading to an erroneous velocity prediction, when this reduction is not included in the calculation. Therefore, the systematic series results also have been used to reanalyse the forward speed effects on stability for all considered hull form variations.

The upright resistance, the heeled resistance, the sideforce and the stability could be expressed in the simple hull form parameters:

$$\frac{L_{WL}}{V_c^{1/3}}, \frac{B_{WL}}{T_c}, \frac{T_c}{T}, \frac{L_{WL}}{B_{WL}}, \frac{B_{WL}}{V_c^{2/3}}, \frac{L_{WL}}{C_B}, \frac{C_p}{\sqrt{gL_{WL}}}.$$

at constant $V/\sqrt{gL_{WL}}$. 
Figure 1: Comparison of IMS and Delft approximations of the residuary resistance with experiments. From [2].

The resulting polynomial expressions may be used for a velocity prediction calculation for a given sailing yacht of known geometry, sailplan and initial stability, assuming that the corresponding sail coefficients are known.

The calculation procedure concerns calm water conditions, assuming that an incident wave system has no influence on the performance of the yacht.

The influence of sea waves with a direction forward of the beam can be estimated when the motions of the yacht due to these waves are known.
The added resistance in waves is related to the damping energy radiated from the oscillating hull. In particular heave- and pitch damping energy is important in this respect, whereas horizontal motions such as sway and yaw can be neglected in this respect [1].

The calculation of the vertical motions and added resistance in waves can be carried out by so-called strip-theory methods.

These simplified methods are limited due to neglect of certain 3-dim effects, in particular in resonance conditions. However, for practical purposes the simplification of the strip theory method is acceptable, at least for analysing purposes. This also applies to the effect of heel angle on the motions in waves.

In general the influence of heel on vertical motions and added resistance is relatively small [1].

The difference of the dynamic response to waves between a light- and medium- or heavy-displacement yacht with comparable length and beam is mainly due to the difference in the natural periods of heave and pitch and the relative damping of these motions.

In general the light displacement yachts have smaller natural pitch and heave periods and larger relative pitch and heave damping.

This causes differences in the added resistance operator which represents the added resistance in regular waves of different length and unit wave height.

In particular there is a shift of the added wave resistance operator to smaller wave-lengths in the case of light-displacement yachts.

When the added resistance response operator for a particular yacht is known, from model experiments or calculation, the added resistance can be determined when the wave spectrum of the considered wave condition is given. The total resistance in waves may be used to carry out a velocity prediction calculation in waves [1].

Directional spreading of wave energy can be included in this procedure, but in view of a lack of data in this respect such a refinement does not seem appropriate.

As a further simplification the wave direction may be set equal to the true wind direction.

With regard to the determination of the added resistance operator for a given yacht it should be remarked that the computing time using strip theory methods is relatively small.

On the other hand it has been shown that the added resistance operator can be very easily expressed by a polynomial expression using only main hull form parameters [3].

In particular for rating purposes such a polynomial expression for the added resistance operator could be useful.

In both the cases standard wave spectra, for instance a Bretschneider formulation using $R_{1/3}$ and $T_1$, can be used to
compute the added resistance, but in principle any measured wave spectrum can be applied. Added resistance and velocity predictions in seaways may serve as to show the importance of hull form, mass and the distribution of mass, with some emphasis on the influence of $L_{WL}/V_c^{1/3}$ and the pitch gyradius ratio $k_r/L_{WL}$ [1, 4].

2. Velocity prediction in calm water

In 1977 the results of model experiments with 9 systematic variations of sailing yacht hull forms were published [5]. The measurements included the determination of the upright resistance, the heeled and induced resistance, the sideforce and the stability. An extension of this research with another series of 12 hull forms was presented in 1981 [6]. All of the 22 hull form variations were based on the sailing yacht Standfast 43 designed by Frans Maas. (Series I). In view of the trend towards light-displacements a further extension of the series with 6 models (Series II) was completed providing the same kind of information as for Series I and published in 1988 [7] and 1991 [8]. These hull form variations were based on a van de Stadt & Partners designed parent form. Finally a third series (Series III) of eleven models has been tested, but only in the upright condition, without leeway. The speed range for Series I is limited to $F_n = 0.45$, but for the Series II and III speeds corresponding to $F_n = 0.75$ have been included. With the parent model of Series I three modifications of the keel span have been tested.

2.1. Main dimensions and form coefficients

The main dimensions of the models 1 - 39, extrapolated to a waterline length $L_{WL} = 10$ meter are given in Table 2, whereas in Table 3 the form coefficients and the longitudinal position of the centre of buoyancy are summarized.

In Table 1 the ranges of some ratio's of main dimensions and form coefficients are given.

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Main dimensions

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The parent body plans for models 1 - 22 and 29 - 39 are depicted in Figure 2. The waterline length of all models of Series I (nrs. 1 - 22) is 1.60 meter; for the Series II and III (nrs. 23 - 28 and nrs. 29 - 39) the waterline length is 2.0 meter.

PARENT MODEL (NO. 1 - 22)

PARENT MODEL (NO. 23 - 39)

Figure 2: Parent models for the Delft Systematic Yacht Hull Series.

All models were tested with the same keel and rudder and consequently with the uniform extrapolation to $L_{WL} = 10$ meters there is a difference in keel span for the Series I on the one hand and the Series II and III on the other hand. For Series I the keel span is 1.37 meter and for Series II and III this is 1.10 meter for the corresponding waterline length $L_{WL} = 10$ meters. The keel and rudder location is given in Figure 3. For the additional keel span variations of model 1 the following cases have been considered for the models 1a, 1b and 1c respectively: 1.25 meter, 1.45 meter and 0.69 meter.
Geometry of keel and rudder

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<th>wetter area m²</th>
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<th>tipchord m</th>
<th>span m</th>
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Figure 3: Position of keel and rudder.
2.2. determination of the hydrodynamic resistance

The total hydrodynamic resistance of a sailing yacht in calm water may be split up in three components:

\[ R_v = R_T + R_i + R_H \]  
(1)

Where:
- \( R_T \) - upright resistance (no leeway)
- \( R_i \) - induced resistance due to the generation of side force
- \( R_H \) - resistance due to heel (no side force)

2.2.1. Upright resistance

The upright resistance is split up in frictional resistance \( R_F \) and residuary resistance \( R_R \).

The frictional resistance is calculated using the 1957 ITTC extrapolator:

\[ C_F = \frac{0.075}{(\log R_n - 2)^2} \quad R_n = \frac{VL}{\nu} \]  
(2)

where the Reynolds number \( R_n \) for the hull is based on \( L = 0.7 \text{ LWL} \). For keel and rudder the mean chord lengths have been used.

It has been considered to use the so called Prohaska form factors in the extrapolation procedure, but the difference in the final result is not significant.

For the analysis of the model experiment results the kinematic viscosity \( \nu \), corresponding to the measured tank water temperature has been used in all cases.

For resistance prediction purposes:

\[ \nu = 1.14 \times 10^{-6} \text{ and } 1.19 \times 10^{-6} \text{ m}^2\text{sec}^{-1} \]

for fresh water and seawater respectively at 15 degrees Celsius may be used.

The wetted surface of the canoe body, without keel and rudder can be approximated by:

\[ S_C = [1.97 + 0.171 \frac{BWL}{T_C}] \times \left[ \frac{0.65}{C_M} \right]^{1/3} \times [\nu_C \times LWL]^{1/2} \]  
(3)

with:
\[ C_M = \frac{\nu_C}{LWL \times BWL \times T_C \times C_p} \]
The frictional resistance follows from:

$$R_F = \frac{1}{2} \rho V^2 (S_C C_{FC} + S_K C_{FK} + S_R C_{FR})$$  \hspace{1cm} (4)$$

where the indices c, k and r refer to respectively the canoe body, the keel and the rudder.

Using a least squares method the residuary resistance of all tested models is expressed in a polynomial expression, using hull form parameters as variables.
For the speed range $Fn = 0.125\,(0.025)\,0.450$ the parameters $C_p$, $LWL/V_c^{1/3}$, LCB and $B_{WL}/T_C$ have been used:

$$\frac{R_R}{\Delta_C} \cdot 10^3 = a_0 + a_1 C_p + a_2 (LCB) + a_3 (B_{WL}/T_C) +$$

$$+ a_4 (LWL/V_c^{1/3}) + a_5 C_p^2 + a_6 C_p \cdot (LWL/V_c^{1/3}) +$$

$$+ a_7 (LCB)^2 + a_8 (LWL/V_c^{1/3})^2 + a_9 (LWL/V_c^{1/3})^3$$  \hspace{1cm} (5)$$

For the speed range $Fn = 0.475\,(0.025)\,0.750$ the polynomial fit is as follows:

$$\frac{R_R}{\Delta_C} \cdot 10^3 = C_0 + C_1 (LWL/B_{WL}) + C_2 (A_w/V_c^{2/3}) + C_3 (LCB) +$$

$$+ C_4 (LWL/B_{WL})^2 + C_5 (LWL/B_{WL}) \cdot (A_w/V_c^{2/3})^3$$  \hspace{1cm} (6)$$

The coefficients a and c are given in the Tables 4 and 5.
It should be noted that $\Delta_C$ is the weight of displacement of the canoe body, without keel and rudder. $V_c$ is the corresponding volume of displacement.
The waterline area $A_w$ may be approximated with sufficient accuracy by:

$$\frac{A_w}{LWL \cdot B_{WL}} = 1.313 C_p + 0.0371 (LWL/V_c^{1/3}) - 0.0857 C_p \cdot (LWL/V_c^{1/3})$$  \hspace{1cm} (7)$$
Table 4

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2.2.2. Induced resistance

The induced resistance coefficient for a lifting surface with an effective aspect ratio \( \text{ARS} \) is given by:

\[
C_{Di} = \frac{C_L^2}{\pi \text{ARS}}
\]  

(8)

Similarly, for the hull, keel and rudder combination the induced resistance, resulting from the generated sideforce \( F_H \) can be written as:

\[
R_i = \frac{1}{\pi \text{ARS}} \frac{F_H^2}{\text{QS}_c}
\]  

(9)

where \( \text{ARS} \) is the effective aspect ratio of the hull, keel and rudder combination, and \( q = \frac{1}{2} \rho V^2 \).
Using the results of the resistance measurements with heel angle and leeway, the induced resistance could be expressed by:

\[ R_i = (C_0 + C_2 \varphi^2) \frac{F_H^2}{qS_C} \]  

(10)

where \( C_0 \) and \( C_2 \) depend on the geometry of the hull, keel and rudder combination.

The expression (10) works well for Series I (nrs. 1 - 22) but for the Series II and III (nrs. 23 - 39) an additional term with the Froude number \( F_n \) was necessary to cope with a significant free surface influence on the induced resistance. Thus:

\[ R_i = (C_0 + C_2 \varphi^2 + C_3 F_n) \frac{F_H^2}{qS_C} \]  

(11)

For Series I a fair agreement between (10) and (11) exists for \( F_n = 0.325 \).

With (9) and (10) we find:

\[ \frac{1}{\pi (C_0 + C_2 \varphi^2)} \]

(12)

We now define an effective draught \( T_E \) with:

\[ \frac{T_E^2}{S_C} \]

, than:

\[ T_E^2 = \frac{S_C}{\pi (C_0 + C_2 \varphi^2)} \]  

(13)

and:

\[ R_i = \frac{F_H^2}{\pi T_E^2 q} \]  

(14)

With the measured \( F_H \) values for models 1 - 28 and model 1a, model 1b and model 1c the effective draughts \( T_E \) have been determined for heel angles 0, 10, 20 and 30 degrees. The relative effective draught \( T_E/T \) appears to be strongly dependent on \( T_C/T \), \( B_{WL}/T_C \) and \( \varphi \).
A satisfactory fit to the experimental data is given by:

\[
\frac{T_E}{T} = A_1 \left( \frac{T_C}{T} \right) + A_2 \left( \frac{T_C}{T} \right)^2 + A_3 \left( \frac{B_{WL}}{T_C} \right)
\]  

(15)

with:

\[
A_1 = 4.080 + 0.0370 \varphi - 4.9830 \varphi^3
\]

\[
A_2 = -4.179 - 0.8090 \varphi + 9.9670 \varphi^3
\]

\[
A_3 = 0.055 - 0.0339 \varphi - 0.0522 \varphi^3
\]

\(\varphi\) in radians.

2.2.3. Resistance due to heel

For each of the models 1 - 28 the resistance due to heel, \(R_H\), has been determined.
It was found that a reasonable approximation of \(R_H\) is given by:

\[
\frac{R_H}{qS_C} = C_H Fn^2 \varphi
\]

(16)

\(\varphi\) in radians.

The \(C_H\) was expressed in the keel and hull parameters \(T_C/T\) and \(B_{WL}/T_C\).

\[
C_H \times 10^3 = 6.747 \left( \frac{T_C}{T} \right) + 2.517 \left( \frac{B_{WL}}{T_C} \right) + 3.710 \left( \frac{B_{WL}}{T_C} \right) \times \left( \frac{T_C}{T} \right)
\]

(17)

The resistance due to heel and side force, the heeled resistance is given by:

\[
R_H + R_H = \frac{F_H^2}{\pi T_E^2 q} + (C_H Fn^2 \varphi)qS_C
\]

(18)

with \(T_E\) and \(C_H\) as shown in (15) and (17).

For \(\varphi > 30\) degrees an extra resistance increase can be included to allow for the influence of deck immersion.
By analogy with the IMS formulation the following expression is used for velocity predictions:

\[ R_\varphi = R_{\varphi 0} \left[ 1 + 0.0004(\varphi - 30)^2 \right] \]  \hfill (19)

\( \varphi \) in degrees.

This results in a resistance increase of 1% and 4% respectively for \( \varphi = 35 \) degrees and \( \varphi = 40 \) degrees.

2.3. Side force as a function of heel and leeway

For the models 1 - 22 (Series I) and model 1c (half keel span) the relation between leeway and side force is approximated by:

\[ \beta = \frac{F_H \cos \varphi}{qS_C} (B_0 + B_2 \varphi^2) \]  \hfill (20)

\( \beta \) and \( \varphi \) in radians

Due to larger \( B_{WL}/T_c \) an additional term depending on the heel angle and the Froude number is necessary for the models 23-28 (Series II) to satisfy the experimental evidence which indicates free surface effects. Thus:

\[ \beta = \frac{F_H \cos \varphi}{qS_C} (B_0 + B_2 \varphi^2) + B_3 \varphi^2 F_n \]  \hfill (21)

If the combination of hull, keel and rudder is considered as a side force (lift) generating element, the "lift" slope will be given by the first two terms of (21):

\[ \frac{F_H \cos \varphi}{\beta qS_C} = \frac{1}{B_0 + B_2 \varphi^2} \]  \hfill (22)

The slope depends on the effective aspect ratio of the underwater part of the hull, keel and rudder, which in this case is related to side force generation.
It was found that the "lift" slope can be expressed with sufficient accuracy by: $T_c/T$ and $T^2/S_c$:

$$\frac{F_H \cos \varphi}{\beta \varphi \frac{S_c}{q}} = b_1 \left( \frac{T^2}{S_c} \right) + b_2 \left( \frac{T^2}{S_c} \right)^2 + b_3 \left( \frac{T_c}{T} \right) + b_4 \left( \frac{T_c}{T} \right)^2 \left( \frac{T^2}{S_c} \right)$$

with:

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(23)

The coefficient $B_3$ in (21) has been determined with the experimental results of models 23 - 28 (Series II):

$$B_3 = 0.0092 \left( \frac{BWL}{T_c} \right)^2 \left( \frac{T}{T_c} \right)$$

(24)

The contribution of the $B_3$ is relatively small, except in the case of very large $BWL/T_c$ and $T/T_c$, such as models 24 and 26. Then there is a certain heel angle at which no side force is generated, which follows from:

$$\beta = B_3 \varphi^2 F_n.$$  

2.4. Stability

The data reduction of the experimental stability data has been carried out as follows, see Figure 4.

$$GN \sin \varphi = GM \sin \varphi + MN \sin \varphi$$

(25)

where $GM$ is the calculated hydrostatic value at $V = 0$. The residuary stability lever can be expressed in: $\varphi$, $F_n$ and $BWL/T_c$:

$$\frac{MN \sin \varphi}{L_{WL}} = (D_2 \varphi F_n + D_3 \varphi^2)$$

(26)
with:

\[ D_2 = -0.0406 + 0.0109 \left( \frac{BWL}{T_c} \right) - 0.00105 \left( \frac{BWL}{T_c} \right)^2 \]

\[ D_3 = +0.0636 - 0.0196 \left( \frac{BWL}{T_c} \right) \]

\( \phi \) in radians

Finally the distance of the centre of lateral resistance to the waterline is given by:

\[ D_4 \times T \]

(27)

with:

\[ D_4 = 0.414 - 0.165 \left( \frac{T_c}{T} \right) \]

Apparently for \( T_c/T \to 0 \) \( D_4 \) approaches the value for an elliptic distribution of the sideforce from the tip of the keel to the waterline.

![Diagram](image)

\[ GN \sin \varphi + (GM + MN) \sin \varphi \]

Figure 4: Definition of residual stability lever MN \( \sin \varphi \).
To show the goodness of fit of the various polynomials as given for resistance, side force and stability some results are given in the Figures 5 - 8.

In Figure 5 the measured and predicted upright resistance for the models 16 and 37 (a heavy- and light-displacement hull) are compared. The typical difference in character of the resistance curve for speeds excluding $F_n = 0.45$ is clearly shown.

In Figure 6 the heeled resistance, predicted with equation (18) is compared with the experimental results for models 16 and 28, and in Figure 7 the generated side force as a function of leeway and heel angle predicted according to equation (21) is compared with the measurements.

Finally a similar comparison has been made for the stability lever at 10, 20 and 30 degrees as a function of the Froude number using equation (25) and (26). The examples include some rather extreme hull forms, but the prediction in all considered cases is satisfactory.

The importance of the length-displacement ratio $L_{WL}/V_C^{1/3}$ and the beam to draught ratio $B_{WL}/T_C$ is clearly shown in the Figures 5 - 8.

In particular the attention is drawn to the loss of stability at forward speed for the wide beam models 31 and 33 as depicted in Figure 8.

Figure 5: Measured and predicted upright resistance.
Figure 6: Measured and predicted heeled resistance.
Figure 7: Measured and predicted sideforce.
Figure 8: Stability lever GN sin φ as a function of Fn. The measured values for Fn = .15, .30 and .45 coincide with the drawn lines.
3. Velocity prediction in waves

The added resistance in waves has an important influence on the performance of a sailing yacht. The oscillatory motion of the yacht generates damping waves, which superimpose on the incident seaways. The damping waves, which are mainly due to pitching and heaving motions, radiate the damping energy. The resulting added resistance is forced by equalizing the work done by the added resistance force and the radiated wave damping energy.

To estimate the relative importance of the added resistance in waves the simple strip theory may be used, as discussed in [7].

The mean added resistance in waves, $R_{AW}$, follows from:

$$ R_{AW} = \frac{1}{\lambda} \int_{0}^{\lambda} \int_{0}^{T_e} b' \nu z^2 \, dx \, dt $$

(28)

where:

- $\lambda$ - wave length
- $t$ - time
- $b'$ - cross sectional damping coefficient, corrected for the forward speed
- $\nu z$ - relative vertical velocity of the considered cross-section with respect to the water.
- $T_e$ - period of wave encounter
- $x_d$ - length ordinate of the hull.

The vertical relative motion $\nu z$ is determined by vectorial summation of heave, pitch and incident wave velocity. The strip theory calculation of motions and added resistance agrees quite well with model experiments, as shown in [7] and [9].

Using the superposition principle the added resistance of a yacht in an irregular seaway can be determined when the added resistance response operator, as well as the wave spectrum are known.

The wave spectrum may be available from actual wave buoy measurements or approximated using visual estimates of the significant wave height $H_{1/3}$ and the average period $T$, in a standard formulation for the wave energy distribution, as given for instance by Bretscheider [7].

With the computational capacity of today's personal computers, the added resistance response operator of a yacht can be easily determined, using strip theory methods, when the
linesplan and the longitudinal distribution of mass are
given.
The mean added resistance $R_{AW}$ in a wave spectrum $S_\omega$ follows
from:

$$
\dot{R}_{AW} = 2 \int_{0}^{\infty} \dot{R}_{AW} * S_\omega(\omega_e) d\omega_e
$$

(29)

where $\omega_e$ is the frequency of encounter.

The added resistance operator $R_{AW}/L_a^2$, or a corresponding
dimensionless presentation, such as $R_{AW}/\rho g L_a^2 L_{WL}$, depends
on the hull geometry, the longitudinal gyradius $k_{yy}$, the wave
period or frequency and the wave direction $\mu_w$.

For all models of the Delft Series the added resistance
operator has been calculated by Reumer for range of Froude
numbers, wave frequencies and wave directions [3].
With a least squares procedure the resulting added resistance
operators could be expressed in a polynomial expression:

$$
\frac{R_{AW}}{L_a^2} = a_1 (L_{WL}/V_C)^{1/3} + a_2 (L_{WL}/V_C)^{1/3})^2 +
+ a_3 (L_{WL}/V_C)^{1/3})^3 + a_4 (L_{WL}/B_{WL}) + a_5 (L_{WL}/B_{WL})^2 +
+ a_6 (B_{WL}/T_C) + a_7 C_p + a_8 C_p^2 + a_9 C_p^3.
$$

(30)

The coefficients $a_i$ are a function of the wave direction,
wave frequency and the Froude number. The calculations have
been carried out for $L_{WL} = 10$ meters and a gyradius $k_{yy}/L_{WL} = 0.25$.
In Figure 9 the result of (30) is compared with a direct
computation, using strip theory, for the models 1 and 25 for
$\mu_w = 165^\circ$ (15° off the bow) and $F_n = 0.25$.

Computed added resistance operators have been used to analyse
the influence of the pitch gyraadius and the displacement-
length ratio on the mean added resistance in an irregular
seaway defined by:

$$
S_\omega = A \omega^{-5} \exp(-B \omega^{-4})
$$

(31)

with:

$$
A = 173 \frac{H_{1/3}}{T^4}
$$

$$
B = 691/T^4
$$
Figure 9: Added resistance operators for models 1 and 25.
The calculated $\bar{R}_{AW}$ for $T_1 = 2$, 4 and 6 seconds, $k_{yy}/L_{WL} = 0.23$, 0.27 and 0.31, and $L_{WL} = 10$ meters is depicted in Figure 10 on a base of:

$$\frac{V_c^{1/3}}{L_{WL}} \cdot \frac{k_{yy}}{L_{WL}},$$

as a function of wave direction.

**WAVE DIRECTION**
- $\nu_w = 100$ degr.
- $\nu_w = 115$ degr.
- $\nu_w = 125$ degr.
- $\nu_w = 135$ degr.

**SPECTRUM:**
- $T_1 = 2$ SBC, $H_{1/3} = 0.50$ M,
- $FN = 0.35$, $L_{WL} = 10$ M.

Figure 10a: Added resistance.
Figure 10b: Added resistance.
Figure 10c: Added resistance.

The eight models 1, 5, 6, 22, 25, 26, 30 and 31 constitute a very large range of hull form variation. Therefore the data in Figure 10 can be regarded to represent the total series with respect to the added resistance in waves. Also the pitch gyroradius range, as chosen, is very wide in particular for the medium- and heavy-displacement hull forms.
For one wave direction $\mu_w = 135$ degrees the added resistance has been plotted on a base of mean wave period ($F_n = 0.35$, $L_{WL} = 10$ meters) - see Figure 11, which shows the importance of the mean wave period or mean wave length for the added resistance in waves, as well as the influence of the pitch gyroradius.

The importance of the added resistance is shown by relating $\bar{R}_{AW}$ to the upright resistance $R_T$, which is 1261 N for hull nr. 1 and 657 N for nr. 26.

![Graph showing added resistance versus mean wave period for different values of $k_{yy}/L_{WL}$](image)

**Figure 11a:** Added resistance versus mean wave period Model 1.
Figure 11b: Added resistance versus mean wave period model 26

The added resistance calculation has been carried out for \( F_n = 0.15, 0.25, 0.35, 0.45 \) and 0.60. The total result is given in dimensionless form to facilitate the use for waterline lengths other than \( L_{WL} = 10 \) meters. To this end the added resistance operator is expressed as \( R_{AW}/\rho g L_{WL} H_{w/3} \) as a function of \( T_1/g/L_{WL}, F_n \) and \( \mu_w \).
The following example illustrates the calculation procedure for a yacht with $L_{WL} = 15$ meters, $L_{WL}/V_{c}^{1/3} = 5.8$, $k_{yy}/L_{WL} = 0.30$ in a wave-spectrum $R_{1/3} = 1.2$ meter, $T_{1} = 3$ seconds and $F_{n} = 0.35$, $\mu_{w} = 135$ degrees, see Figure 12.
In this case $V_{c}^{1/3}/L_{WL} \cdot k_{yy}/L_{WL} = 0.052$ and $R_{AW}/\rho g L_{WL} R_{1/3}^{2} = 0.0079$.
Thus, $R_{AW} = 0.0079 \times 1025 \times 9.81 \times 15 \times 1.2^2 = 1716$ N.

Figure 12: Example $R_{AW}$ calculation.

The total result of the added resistance calculation has been approximated by a least squares procedure to enable the use of the data for a velocity predition procedure.
It should be remarked that the added resistance calculation, as presented, is an approximation, based on linear strip theory.
It is assumed that the wave direction is equal to true wind direction.
Also the influence of the heel angle is not included. As shown in [7] this may be acceptable in many cases, but some discrepancies compared with experimental results have been observed. In general strip theory methods are a reasonable tool to estimate ship motions and added resistance in seaways. More accurate 3-dimensional methods are now available, but in view of other uncertainties such as non-linearities and the description of the irregular sea surface, the increased complexity of such methods seems not justified.

However the present method, as described in the paper is thought to be adequate for design and rating purposes.

4. References


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